## Formal Geometry: Chapter 6 Review Worksheet



For the entire worksheet: Round to 2 decimal places unless exact answers are specified.

1. Which of the following statements are true? Choose all that apply.
A) A parallelogram with congruent diagonals is a rhombus.
B) A parallelogram with one right angle is a square.
(C) A rhombus that is also a rectangle is a square.
(D) A parallelogram with diagonals that bisect the angles is a rhombus.

Sometimes it helps to think in pictures when you have questions like this.
A) Diagonals are congruent in a rectangle, which is a parallelogram, but is not necessarily a rhombus. FALSE
B) A parallelogram with one right angle is a rectangle, which is not necessarily a square. FALSE

C) As can be seen in the quadrilateral tree above, if a figure is both a rhombus and a rectangle, then it must be a square. TRUE
D) Each diagonal in a rhombus bisects a pair of opposite angles. This is not true for all parallelograms. TRUE


Selections C and D are true. See also problems 16 - 19.
2. The diagonals of a rhombus are 14 in and 48 in . Find the side length of the rhombus.

Lets take one triangle from the inside of the rhombus shown to the right. See below.

We know that the diagonals are

perpendicular, so we have a right

triangle. The two red sides of the triangle are half of the length of the diagonals from which they come.

We have sides, then, of $a=14 \div 2=7$ and $b=48 \div 2=24$.
It remains for us to calculate the value of $c$. Let's use the Pythagorean Theorem:

$$
\begin{aligned}
& c^{2}=a^{2}+b^{2} \\
& c^{2}=7^{2}+24^{2} \\
& c^{2}=49+576=625 \\
& c=25 \text { inches (remember to use units in the answer because they are in the statement } \\
& \text { of the problem) }
\end{aligned}
$$

3. Find the value of each variable in the parallelogram shown below.

In a parallelogram, opposite angles are congruent and consecutive angles are supplementary. This gives us:

$$
\begin{aligned}
& x^{2}+5 x=2 x+28 \\
& x^{2}+3 x-28=0 \\
& (x+7)(x-4)=0 \\
& x=-7 \text { or } 4
\end{aligned}
$$



If $x=-7$, then the angles involved are equal to $(2 x+28)^{\circ}=(2(-7)+28)^{\circ}=14^{\circ}$. If $x=4$, then the angles involved are equal to $(2 x+28)^{\circ}=(2(4)+28)^{\circ}=36^{\circ}$.

Both of these angle values are possible, so we have two cases:
If $x=-7$, then the angles involved are $14^{\circ}$. Since consecutive angles are supplementary, $14+8 y=180$.

$$
\begin{aligned}
8 y & =166 \\
y & =20.75 \text { and the solution is: } x=-7, y=20.75
\end{aligned}
$$

If $x=4$, then the angles involved are $36^{\circ}$. Since consecutive angles are supplementary, $36+8 y=180$.

$$
\begin{aligned}
8 y & =144 \\
y & =18 \text { and the solution is: } \boldsymbol{x}=\mathbf{4}, \boldsymbol{y}=\mathbf{1 8}
\end{aligned}
$$

Both solutions are valid because they both result in positive angle values.

## For \#4, Find the value of $\boldsymbol{x}$ and $\boldsymbol{y}$ so that the quadrilateral is a parallelogram.

4. In order for the figure to be a parallelogram, opposite sides must be congruent. So,

$$
\begin{aligned}
& \text { Top and bottom sides: } \\
& \begin{array}{c}
\text { Left and right sides: } \\
2 x+4 y=5 y+x \\
x=y \\
3 x+3 y=12 \\
x+y=4 \\
y=4-x
\end{array} \\
& \text { Combine above results: } \\
& x=4-x \\
& 2 x=4 \\
& x=2
\end{aligned} \quad \begin{gathered}
x=y \\
2=y
\end{gathered}
$$



Solution: $x=2, y=2$
5. Find the diagonal of a square with a perimeter of 32 inches. Exact answer only. Let:

- $s$ be the length of a side of the square.
- $d$ be the length of the diagonal of a square.

The perimeter of a square is four times the length of one side. So, we have the length of a side, $s=32 \div 4=8$ inches.

Using the Pythagorean Theorem, we have:

$$
\begin{aligned}
& d^{2}=a^{2}+b^{2} \\
& d^{2}=8^{2}+8^{2} \\
& d^{2}=64+64=2 \cdot 64
\end{aligned}
$$



8
$c=\sqrt{2 \cdot 64}=\sqrt{2} \cdot \sqrt{64}=8 \sqrt{2}$ inches (remember to use units in the answer because they are in the statement of the problem)
6. Given the following vertices, determine whether quadrilateral BEFG is a parallelogram, rhombus, a rectangle, or a square. List all that apply. Show all evidence. B(1, 2), E(-2, -1), $\mathrm{F}(2,-5), \mathrm{G}(5,-2)$

See the figure to the right. It looks like a rectangle, which would also make it a parallelogram, but not a rhombus or square. Let's check lengths.

- $B E=\sqrt{(1-(-2))^{2}+(2-(-1))^{2}}$

$$
=\sqrt{9+9}=\sqrt{2 \cdot 9}=3 \sqrt{2}
$$

- $G F=\sqrt{(5-2)^{2}+(-2-(-5))^{2}}$


$$
=\sqrt{9+9}=\sqrt{2 \cdot 9}=3 \sqrt{2}
$$

- $B G=\sqrt{(1-5)^{2}+(2-(-2))^{2}}$

$$
=\sqrt{16+16}=\sqrt{2 \cdot 16}=4 \sqrt{2}
$$

- $E F=\sqrt{(2-(-2))^{2}+(-5-(-1))^{2}}$

$$
=\sqrt{16+16}=\sqrt{2 \cdot 16}=4 \sqrt{2}
$$

Conclude: BEFG has two pair of opposite sides that are equal in length. The lengths of the pairs, however, are different ( $3 \sqrt{2}$ and $4 \sqrt{2}$ ). This is true for a parallelogram and a rectangle, but rules out the rhombus and the square (which have all sides equal).

Let's check slopes.

- $m_{B E}=\frac{2-(-1)}{1-(-2)}=\frac{3}{3}=1$
- $m_{B G}=\frac{-2-2}{5-1}=\frac{-4}{4}=-1$
- $m_{F G}=\frac{-2-(-5)}{5-2}=\frac{3}{3}=1$
- $m_{F E}=\frac{-5-(-1)}{2-(-2)}=\frac{-4}{4}=-1$

Conclude: Each pair of adjacent segments is perpendicular because the slopes of the segments are negative reciprocals.

Therefore, BEFG has four right angles, so BEFG is a rectangle. Since every rectangle is also a parallelogram, BEFG is also a parallelogram. As noted above, BEFG is neither a rhombus nor a square.
7. If the diagonals of a rhombus are 24 and 10 . Find the perimeter of the rhombus.

Lets take one triangle from the inside of the rhombus shown to the right. See below.

We know that the diagonals are

perpendicular, so we have a right

triangle. The two red sides of the triangle are half of the length of the diagonals from which they come.

We have sides, then of $a=10 \div 2=5$ and $b=24 \div 2=12$.
Next, let's calculate the value of $c$. Let's use the Pythagorean Theorem:

$$
\begin{aligned}
& c^{2}=a^{2}+b^{2} \\
& c^{2}=5^{2}+12^{2} \\
& c^{2}=25+144=169 \\
& c=13
\end{aligned}
$$

The perimeter of a rhombus is four times the length of one side.

$$
P=4 c=4(13)=52 \text { (no units are given) }
$$

8. Find the perimeter of the rhombus. Exact answer.

Remember not to trust the diagram. In the figure to the right, the angle labeled $60^{\circ}$ looks a good bit smaller than $60^{\circ}$. Let's look at the information from the Geometry Handbook on special triangles (to the left). Notice

 that the side of the triangle opposite the $60^{\circ}$ angle is $\sqrt{3} \cdot$ (short side), and the hypotenuse is of the triangle is $2 \cdot$ (short side).

Now, we can construct the triangle to the right (proportioned properly) to obtain the length of the hypotenuse, and then the
 perimeter.

$$
c=8 \sqrt{3}
$$

The perimeter of a rhombus is four times the length of one side.

$$
P=4 c=4(8 \sqrt{3})=32 \sqrt{3} \text { (no units are given) }
$$

9) Find $x$ and $y$ from the rhombus to the right.

In a rhombus, the diagonals intersect at right angles, so:

$$
\begin{aligned}
& 5 x+5 y=90 \\
& x+y=18 \\
& y=18-x
\end{aligned}
$$



In a rhombus, the sides have the same length, so:

$$
\begin{aligned}
& 6 x-23=2 y-3 \\
& 6 x-20=2 y \\
& 3 x-10=y
\end{aligned}
$$

Combining the two equations:

$$
\begin{aligned}
& 3 x-10=18-x \\
& 4 x=28 \\
& x=7
\end{aligned} \quad \square \begin{aligned}
& y=18-x \\
& y=18-7 \\
& y=\mathbf{1 1}
\end{aligned}
$$

10) Find $x$ and $y$ if $A D \cong B C$ and $A B C D$ is a parallelogram.
$\overline{A D}$ and $\overline{B C}$ are diagonals. Since the diagonals are congruent and $A B C D$ is a parallelogram, we conclude that ABCD is a rectangle. Therefore, all four interior angles measure $90^{\circ}$. Then,

$$
\begin{array}{l|l|l}
14 y+20=90 & 12 x-42=90 & 7 y+55=90 \\
14 y=70 & 12 x=132 & 7 y=35 \\
\boldsymbol{y}=\mathbf{5} & \boldsymbol{x}=\mathbf{1 1} & y=5
\end{array}
$$



Fortunately, the first and third column result in the same value for $y$. If this were not the case, we would say this problem is overdetermined, and there would be no solution for $y$.
11. $J K L M$ is a rhombus. If $m \angle J M L=30^{\circ}$ and $J K=7 \mathrm{in}$, then find the value of each requested measure.

The diagram is redrawn at the right based on what we are given and what we know about rhombuses.

JKLM is a rhombus, so its diagonals are perpendicular. This makes all four of the center angles $90^{\circ}$.


In a rhombus, the diagonals bisect the inner angles.
$m \angle J M L=30^{\circ}$, so all four of the small angles shown measure $30^{\circ} \div 2=15^{\circ}$.
In a rhombus, all four sides are congruent. If $\mathrm{JK}=7$, all four sides have length 7 .
a) $m \angle K L J$
$\angle K L J$ is part of $\triangle \mathrm{KLC}$, in which the inner angles must add to $180^{\circ}$. Therefore,
$m \angle \mathrm{KLJ}=180^{\circ}-90^{\circ}-15^{\circ}=75^{\circ}$
b) MK, to the nearest hundredth.

We will need a little Trigonometry to determine MK.
Extract the upper left triangle from the above diagram.

$$
\begin{aligned}
& \cos 15^{\circ}=\frac{a}{7} \\
& a=7 \cdot \cos 15^{\circ}=6.7615
\end{aligned}
$$



MK is double the length of $a$ in the above diagram.

$$
\mathrm{MK}=2 a=2 \cdot 6.7615 \sim 13.52
$$

12. What is the measure of $H J$ in Parallelogram $F G H J$, given the following:

$$
\begin{array}{ll}
F G=x+7 & m \angle F=46^{\circ} \\
G H=5 x+3 & m \angle H=(3 x+10)^{\circ}
\end{array}
$$

A. $\mathrm{HJ}=63$
(B.) $\mathrm{HJ}=19$
C. $\mathrm{HJ}=12$
D. $\mathrm{HJ}=8$

Parallelogram FGHJ has been redrawn on the right, adding the information provided in this problem.

First, opposite angles in a parallelogram have equal measure, so we can find $x$ as follows:


$$
\begin{aligned}
& 3 x+10=46 \\
& 3 x=36 \\
& x=12
\end{aligned}
$$

Then, opposite sides have the same length, so

$$
\mathbf{H J}=\mathrm{FG}=x+7=12+7=\mathbf{1 9} \quad \text { Answer } \mathbf{B}
$$

13. Garrett is trying to prove that quadrilateral ACDB is a parallelogram. Which of the following methods would be valid? Choose all that apply.

Show that both pairs of opposite angles are congruent.
B) Show that both pairs of opposite sides are congruent.
C) Show that the diagonals bisect each other.
D) Show that one pair of opposite sides is both parallel and congruent.

All four of these are sufficient to prove that quadrilateral $A B C D$ is a parallelogram. If you want to convince yourself of this, try to draw a quadrilateral with the properties given that is not a parallelogram. Also, see the chart on the Characteristics of Parallelograms in the Geometry Handbook.

Answer A, B, C, D
14. Find the length of the longest diagonal in a rhombus with a perimeter of 32 and one angle of 74 degrees.

The diagram is redrawn at the right based on what we are given and what we know about rhombuses.

JKLM is a rhombus, so its diagonals are perpendicular. This makes all four of the center angles $90^{\circ}$.

In a rhombus, the diagonals bisect the inner angles.

$m \angle \mathrm{JML}=74^{\circ}$, so $m \angle \mathrm{JMK}=37^{\circ}$.
In a rhombus, all four sides are congruent. If $\mathrm{P}=32$, each side has length $32 \div 4=8$.
The longest diagonal bisects the smallest angle, so we want the measure of MK. We will need a little Trigonometry for this purpose.

Extract the upper left triangle from the above diagram.

$$
\begin{aligned}
& \cos 37^{\circ}=\frac{a}{8} \\
& a=8 \cdot \cos 37^{\circ}=6.38908
\end{aligned}
$$



MK is double the length of $a$ in the above diagram.

$$
\mathrm{MK}=2 a=2 \cdot 6.38908 \sim 12.78
$$

15. Parallelogram ABCD has the following known vertices: $\mathrm{A}(-2,5) ; \mathrm{B}(-4,1) ; \mathrm{C}(1,2)$. Find the coordinates of vertex $D$.

To find the fourth coordinate, graph the initial three and connect them, as shown to the right. Then, vertex $\mathbf{D}$ must, in this case, be to the upper right of C to form a parallelogram.

To find where $\mathbf{D}$ must be, 1) determine how to move from $B$ to $A$, and 2) move the same way to get from $C$ to $D$.


To get from B to A in the graph, we must move right 2 and up 4 . Note that right 2 and up 4 can be represented by the vector $\langle 2,4\rangle$.

When we do this from C, we get:

$$
\begin{array}{r}
C(1,2) \\
+\langle 2,4\rangle \\
\hline \mathbf{D}(3,6)
\end{array}
$$



For \#16 - 19: Is each statement always, sometimes, or never true?
Three tips: 1. Draw. 2. Draw. 3. Draw.
16. If a quadrilateral has congruent diagonals, then it is a rectangle.

S - sometimes. Rectangles have congruent diagonals, but it is possible to construct a quadrilateral with congruent diagonals that is not a rectangle. See the figure to the right, which has congruent diagonals.

17. If a quadrilateral is a square, then it is a paralellogram.

A - always. This can be seen in the quadrilateral tree on page 1 of this document. A square is a parallelogram with four congruent $\left(90^{\circ}\right)$ angles and four congruent sides.
18. If a quadrilateral is a rhombus, then its diagonals create 4 congruent isosceles triangles.

S - sometimes. A rhombus and its diagonals create four congruent triangles, but they are not necessarily isosceles, as shown in the figure to the right.


By squeezing the rhombus shown above in from the sides, we could create a rhombus with four congruent isosceles triangles (see the figure to the left). If we did this we would end up with a square.
19. If a quadrilateral is a kite, then it's diagonals are perpendicular.

A - always. A kite's diagonals are always perpendicular. From the Geometry Handbook:

## Facts about a Kite

To prove a quadrilateral is a kite, prove:

- It has two pair of congruent sides.
- Opposite sides are not congruent.

Also, if a quadrilateral is a kite, then:

- Its diagonals are perpendicular

- It has exactly one pair of congruent opposite angles.

20. Find the Perimeter of the kite.

The figure to the right has been redrawn with points identified for reference purposes. Notice that $\mathrm{BM}=5$.

$$
\Delta \mathrm{BMC} \text { is a } 45^{\circ}-45^{\circ}-90^{\circ} \text { triangle, so } \mathrm{BC}=5 \sqrt{2}
$$

$$
\Delta \mathrm{DMC} \cong \Delta \mathrm{BMC}, \text { so } \mathrm{DC}=5 \sqrt{2}
$$


$\triangle \mathrm{BMA}$ is a $30^{\circ}-60^{\circ}-90^{\circ}$ triangle, with $\overline{\mathrm{BM}}$ the short side, so $\mathrm{BA}=10$
$\triangle \mathrm{DMA} \cong \triangle \mathrm{BMA}$, so $\mathrm{DA}=10$
Finally, $P=5 \sqrt{2}+5 \sqrt{2}+10+10=\mathbf{2 0}+\mathbf{1 0} \sqrt{\mathbf{2}}$ units
21. Given a trapeziod with $\mathrm{b}_{1}=2 x^{2}-14 \mathrm{~cm}, \mathrm{~b}_{2}=8 x+4 \mathrm{~cm}$, and $\mathrm{m}=5 x+15 \mathrm{~cm}$, find m . $m$ is the average (mean) of $b_{1}$ and $b_{2}$. So,

$$
\begin{aligned}
& m=\frac{b_{1}+b_{2}}{2} \\
& 5 x+15=\frac{\left(2 x^{2}-14\right)+(8 x+4)}{2} \\
& 10 x+30=\left(2 x^{2}-14\right)+(8 x+4)
\end{aligned}
$$



Next, collect terms, all on one side of the $=$ sign.

$$
\begin{aligned}
& 0=2 x^{2}-2 x-40 \\
& 0=\left(x^{2}-x-20\right) \\
& 0=(x-5)(x+4) \\
& x=5,-4 \longrightarrow
\end{aligned} \begin{aligned}
& x \text { cannot be }-4 \text { because that would make } \\
& b_{2} \text { and } m \text { negative, but negative lengths are } \\
& \text { not allowed. Finally, } x=5, \text { so } \\
& \\
& m=5 x+15=5(5)+15=40 \mathrm{~cm}
\end{aligned}
$$

22. Find the area of the isosceles trapezoid. The figure to the right has been redrawn with points and lengths identified for the calculation below. $m$ is the midsegment of the trapezoid.


In the figure:

- $\triangle \mathrm{ABF} \cong \triangle \mathrm{DCE}$, both are right triangles.
- BCEF is a rectangle.

We want the total area of the trapezoid. The formula for this is:

$$
\begin{aligned}
& \text { Area }=\frac{b_{1}+b_{2}}{2} \cdot h=m \cdot h \\
& m=\frac{7+19}{2}=13
\end{aligned}
$$

$h$ is determined using the $30^{\circ}-60^{\circ}-90^{\circ}(1: \sqrt{3}: 2$ proportions) triangle $\triangle \mathrm{ABF}$. $a$ is the length of the long side of $\triangle \mathrm{ABF}$.

$$
\begin{aligned}
& a=\frac{19-7}{2}=6 \\
& h=\frac{a}{\sqrt{3}}=\frac{6}{\sqrt{3}}=2 \sqrt{3}
\end{aligned}
$$

Finally, Area $=13 \cdot 2 \sqrt{3}=26 \sqrt{3}$ units $^{2}$

